Math 342: Homework 2

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Documentation: I used ChatGPT solely for looking up Latex commands. The main Homework 2 MatLab script and all required dependencies are located in the Homework 2 folder found here: <https://github.com/Connor-Lemons/Emmons-Math-342>. No other resources used.

Problem 1 (1):

Use the Bisection method to find for on .

To begin, set . Calculating the value of at each of these gives:

Because has the same sign as , set and . This gives . Calculating the value of at each of these gives:

Because has the same sign as , set and . This gives .

Problem 2 (14):

Find an approximation to correct to within using the Bisection Algorithm.

Consider the function . Note that the positive root to this function is the value of . Thus, using the Bisection method to find the roots of allows the approximation of . If the error between the approximation and the actual value is to be less than , then Theorem 2.1 can be used to find the bound on the number of iterations necessary to achieve such accuracy. Use the interval , which gives and .

Thus, the first value produced by the Bisection method which is guaranteed to satisfy the accuracy requirement is . Performing the Bisection method for this problem reveals that the accuracy specification is met by .

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| 1 | 1 | 2 | 1.5 | -0.75 |
| 2 | 1.5 | 2 | 1.75 | 0.0625 |
| 3 | 1.5 | 1.75 | 1.625 | -0.3594 |
| 4 | 1.625 | 1.75 | 1.6875 | -0.1523 |
| 5 | 1.6875 | 1.75 | 1.7188 | -0.0459 |
| 6 | 1.7188 | 1.75 | 1.7344 | 0.008057 |
| 7 | 1.7188 | 1.7344 | 1.7266 | -0.01898 |
| 8 | 1.7266 | 1.7344 | 1.7305 | -0.005478 |
| 9 | 1.7305 | 1.7344 | 1.7324 | 0.001286 |
| 10 | 1.7305 | 1.7324 | 1.7314 | -0.002097 |
| 11 | 1.7314 | 1.7324 | 1.7319 | -0.000406 |
| 12 | 1.7319 | 1.7324 | 1.7322 | 0.0004397 |
| 13 | 1.7319 | 1.7322 | 1.7321 | 0.00001682 |

The code and output can be found at the end of this document.

Problem 3 (10):

Use Theorem 2.3 to show that has a unique fixed point on .

Because is a continuously decreasing function, as long as the endpoints are within a given interval, the rest of the function is guaranteed to be within that interval. Evaluating at each end on the interval gives:

Given that for all , is guaranteed to have at least one fixed point in the interval .

Again, because is a continuously decreasing function, and because has a continuously decreasing derivative, the largest value of the derivative of will occur at the leftmost endpoint.

Because there is a positive constant for which for all , Theorem 2.3 says that there is exactly one fixed point in the given interval.

By Corollary 2.5, the error bound for fixed point iteration is given by:

Note that the bound on the error will be minimized when is chosen exactly between and . Applying this to the problem with a desired accuracy of gives:

From the previous analysis of the derivative of on the given interval, a suitable value of is . Solving the above inequality for gives:

This means that the is the first approximation that is guaranteed to meet the accuracy requirements. Performing fixed point iteration reveals that meets the accuracy requirement of .

|  |  |  |
| --- | --- | --- |
|  |  |  |
| 1 | 0.62996 | 0.036706 |
| 2 | 0.64619 | 0.016234 |
| 3 | 0.63896 | 0.0072304 |
| 4 | 0.64217 | 0.0032103 |
| 5 | 0.64075 | 0.0014274 |
| 6 | 0.64138 | 0.00063427 |
| 7 | 0.6411 | 0.00028192 |
| 8 | 0.64122 | 0.00012529 |
| 9 | 0.64117 | 0.000055684 |

The code and output can be found at the end of this document.

Problem 4 (1):

Let and . Use Newton’s method to find .

Problem 5 (12a):

Use both Newton’s Method ( is the interval midpoint) and Secant Method ( and are interval endpoints) to find solutions with accuracy for:

Newton’s Method on :

|  |  |  |
| --- | --- | --- |
|  |  |  |
| 1 | 1.4067 | 0.09327906 |
| 2 | 1.41237 | 0.005649022 |
| 3 | 1.412391 | 2.121447e-05 |
| 4 | 1.412391 | 2.988791e-10 |

Secant Method on :

|  |  |  |
| --- | --- | --- |
|  |  |  |
| 2 | 1.590616 | 0.4093839 |
| 3 | 1.284548 | 0.3060683 |
| 4 | 1.427966 | 0.1434183 |
| 5 | 1.413635 | 0.01433147 |
| 6 | 1.412378 | 0.001256454 |
| 7 | 1.412391 | 1.299672e-05 |
| 8 | 1.412391 | 1.073098e-08 |

Newton’s Method on :

|  |  |  |
| --- | --- | --- |
|  |  |  |
| 1 | 3.059167 | 0.05916737 |
| 2 | 3.057106 | 0.002061319 |
| 3 | 3.057104 | 2.504693e-06 |
| 4 | 3.057104 | 3.698235e-12 |

Secant Method on :

|  |  |  |
| --- | --- | --- |
|  |  |  |
| 2 | 2.419219 | 1.580781 |
| 3 | 2.75604 | 0.3368211 |
| 4 | 3.317023 | 0.5609828 |
| 5 | 3.009769 | 0.3072535 |
| 6 | 3.050671 | 0.04090175 |
| 7 | 3.057289 | 0.006618332 |
| 8 | 3.057103 | 0.0001861977 |
| 9 | 3.057104 | 7.059887e-07 |
| 10 | 3.057104 | 7.719825e-11 |

The code and output can be found at the end of this document.

Problem 6 (4b):

Use Mueller’s Method to find the zeros of with an accuracy of .

Looking at a graph of the function, there are likely real zeros around and . Choosing and should produce these zeros, and indeed running Mueller’s Method with these initial conditions produces and respectively.

The inflection point just to the right of indicates a likely imaginary root at this location, so choose for Mueller’s Method. This produces , which means that is also a root.

Mueller’s Method for

|  |  |  |
| --- | --- | --- |
|  |  |  |
| 3 | -3.8366 | 51.588 |
| 4 | -3.5245 | -3.5838 |
| 5 | -3.5478 | -0.058736 |
| 6 | -3.5482 | -4.3176e-05 |
| 7 | -3.5482 | 4.1489e-11 |

Mueller’s Method for

|  |  |  |
| --- | --- | --- |
|  |  |  |
| 3 | 4.3964 -0.56232i | -22.392 |
| 4 | 3.8375 -0.59332i | -70.125 |
| 5 | 4.4484 -0.099854i | 8.445 |
| 6 | 4.3801 -0.0058419i | -0.13351 |
| 7 | 4.3811 -6.0609e-05i | -0.0030177 |
| 8 | 4.3811 -5.4476e-09i | 4.1537e-08 |
| 9 | 4.3811 -2.1934e-16i | -7.1054e-14 |

Mueller’s Method for

|  |  |  |
| --- | --- | --- |
|  |  |  |
| 3 | -0.7047 +0 i | -56.288 |
| 4 | 0.65918 +3.214 i | 209.54 |
| 5 | 0.27268 +0.53586i | -32.695 |
| 6 | 0.38851 +1.3029 i | -10.019 |
| 7 | 0.542 +1.4709 i | -1.2194 |
| 8 | 0.58457 +1.4936 i | -0.035052 |
| 9 | 0.58356 +1.4942 i | 5.8656e-05 |
| 10 | 0.58356 +1.4942 i | -3.4301e-10 |

The code and output can be found at the end of this document.

Problem 7 (6):

Show that the sequence converges linearly to and determine the which satisfies .

Consider the limit , which simplifies to . This limit evaluates to 1, which means that the original sequence converges to of order 1 and with asymptotic error constant 1. In order to find such that , simply plug in and solve for .